A SWITCHING APPROACH FOR PERFECT STATE TRANSFER OVER A SCALABLE NETWORK ARCHITECTURE WITH SUPERCONDUCTING QUBITS

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An Important Problem

It is often required in quantum information processing (QIP), to transfer a quantum state from one site to another. It is known that the problem of quantum state transfer with 100% fidelity, called perfect state transfer (PST) can be studied as a problem in graph theory. Therefore an architecture of quantum computation (QC) can be designed purely in terms of graph networks. It has been found that all hypercubes ($\mathcal{Q}_n$) as well as the Cartesian product of path graph P_3 allow PST between their antipodal vertices in time $t_0 = \pi/2$ and $\pi/\sqrt{2}$ respectively, independent of the order n [1, 2]. However, one issue is that PST is not possible from any vertex to any other vertex. Another search could be from a graph that allows any number of qubits and supports PST. Here, we address both of these problems using a technique of switching of edges. We also propose a physical realization model for our architecture with superconducting qubits with tunable couplings.

Proposal for PST in hypercubes (all vertices)

The probability for getting the quantum state localised at the vertex v at time t is given by $|\langle y|\psi(t)\rangle|^2$. Graph G has a PST from vertex x to vertex y at finite time t_0 if

$$|\langle y|\exp(-it_0 A(G))|x\rangle| = 1 \tag{1}$$

where $A(G)$ is the adjacency matrix of G. We propose the switching of sub-hypercubes ($\mathcal{Q}_d$) starting with a hypercube $\mathcal{Q}_n$ (with $|\mathcal{V}_n| = k = 2^n$) by the construction below [4].

Construction: Let $x = (x_1, \ldots, x_n)$, $y = (y_1, \ldots, y_n) \in \mathcal{V}_n$, the vertex set of $\mathcal{Q}_n$. Suppose $d = |\{i : x_i \neq y_i, i = 1, \ldots, n\}|$. Then the unique induced sub-hypercube $\mathcal{Q}_d$ of $\mathcal{Q}_n$ with x, y as antipodal vertices of $\mathcal{Q}_d$ is given by the set of vertices

$$\begin{aligned}\mathcal{V}_d = \{&z = (z_1, \ldots, z_n) \in \mathcal{V}_n : z_i = x_i, \text{ if } x_i = y_i, \text{ and} \\ &z_i \in \{0,1\} \text{ otherwise}, i = 1, \ldots, n\}.\end{aligned} \tag{2}$$

Once the list of such vertices is identified using a quantum or classical memory, a switching technique is proposed to put in place to create an induced sub-hypercube $\mathcal{Q}_d$ of $\mathcal{Q}_n$ such that x, y are antipodal vertices of $\mathcal{Q}_d$. The switching technique involves tuning off the coupling strength of all the edges in $\mathcal{Q}_n$ that do not belong to the induced sub-hypercube $\mathcal{Q}_d$. Indeed, once the vertex set $\mathcal{V}_d$ is determined, deactivate all the couplings that incident to any vertex in $\mathcal{V}_n \setminus \mathcal{V}_d$. This can be accomplished in polynomial time with the aid of classical or quantum memory.

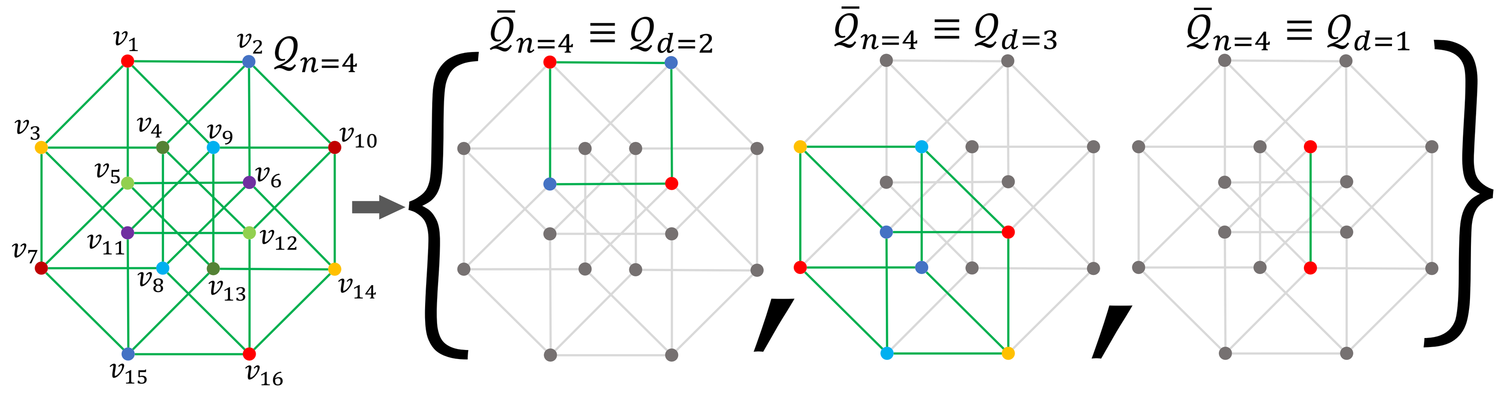

Fig. 1: Visual representation of the switching process. Green represents switched-on edges and gray indicates switched-off edges. Isolated vertices are in dark-gray. Any hypercube $\mathcal{Q}_n$ (here $n = 4$), under switching can realise various embedded sub-hypercubes ($\bar{\mathcal{Q}}_n$ effectively identical to $\mathcal{Q}_d$ for PST dynamics).

PST for any number of qubits ($2^n < k \leq 2^{n+1}$)

We propose a network that enables PST for any number of qubits k, with $2^n < k \leq 2^{n+1}$ for any n. PST in hypercubes is optimal, that is, one-step task in minimal time t_0. For a general k, therefore, the next possible optimal time for PST is a two-step PST at most. We present such a graph construction which performs PST in maximum of two steps, with a total time $2t_0$ [3]. Therefore, now the PST follows in two steps as

$$x \longrightarrow z \longrightarrow y \tag{3}$$

with an intermediate node z. It can proved by partitioning the vertices as $k = 2^n + m$, where additional nodes need to be connected to an existing hypercube. If we follow the binary labeling in sequence for all vertices (with the length $n+1$), then an interesting property arises. If the desired vertices belong to this complete hypercube of 2^n vertices, then a PST is possible. However, if one vertex belongs to the complete hypercube and the other in the additional nodes, the PST can be performed by a 2-step PST process in time $2t_0$. And the same argument follows for all partitions within this set of m qubits. Therefore, a PST is possible between any qubits for any arbitrary number of qubits in a maximum time of $2t_0$. And hence, the scalability of this protocol.

The graph for this PST scheme can be understood with an example. Adapt the labelling of $\mathcal{Q}_{n+1}$ and start with the vertex $v_1 = (000\ldots0)$ - $n+1$ times. Then do the binary addition with $(000\ldots01)$ for the next vertex v_2 and so on. Therefore, the next vertex is $v_2 = (000\ldots01)$, and so on until v_k. Connect the vertices with edges which have a Hamming distance of unity. Then it can be seen from results in [3] that there always exists a special vertex z such that PST can be performed from x to z to y, if not directly between x and y.

Physical realization with superconducting qubits

Our task is to show support for switching of the hypercube $\mathcal{Q}_n$ to $\bar{\mathcal{Q}}_n \equiv \mathcal{Q}_d$ as desired for any pair of chosen vertices of $\mathcal{Q}_n$ for the task of PST. Physical key requirements of our architecture are the following: (a) n nearest-neighbour (NN) interactions to realize any general $\mathcal{Q}_n$ hypercube, (b) Since distant qubits are connected, implementation is not possible in planar integration, $\mathcal{Q}_4$ on wards. Three-dimensional (3D) integration is needed, (c) Tunable (switchable) edges as couplings for each pair of nodes (qubits), and (d) High fidelity control over the processor.

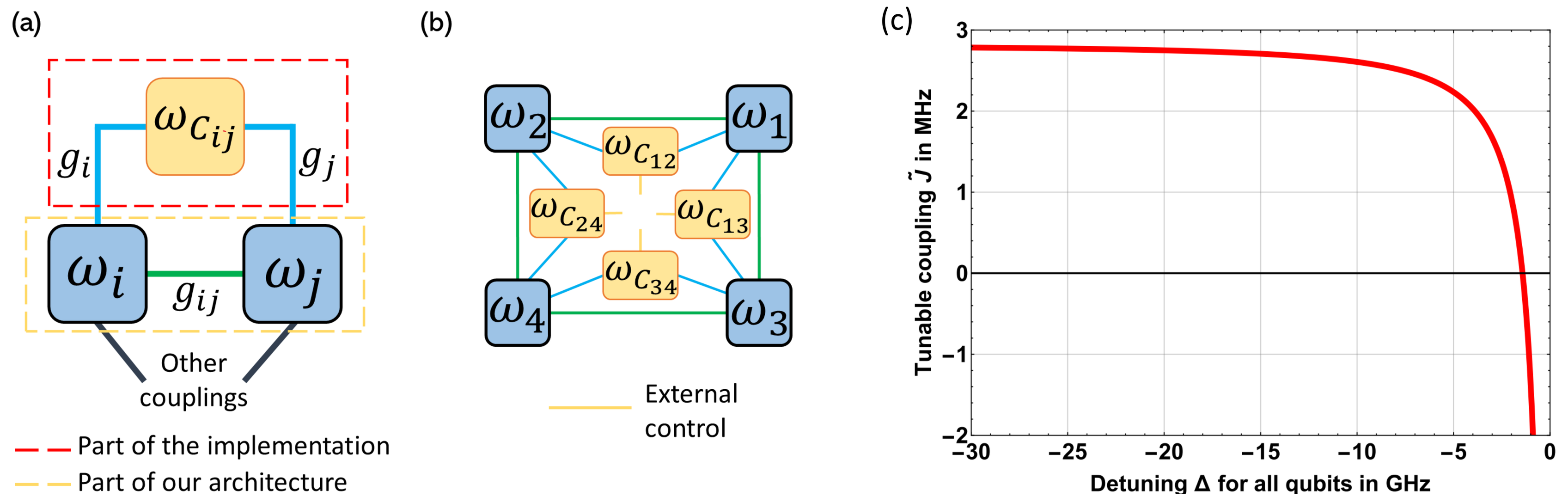

Fig. 2: (a) Couplings involved between a pair of interacting qubits, forming an edge in the hypercube $\mathcal{Q}_n$. (b) Network of four qubits forming $\mathcal{Q}_2$. Each ancillary coupler is associated with every edge which is controlled in the experiment. (c) Variation of the dynamic tunable coupling $\tilde{J}$ w.r.t. the detuning Δ for each qubit. There exists a cutoff value which is always guaranteed.

We consider a general system that consists of 2^n qubits for $\mathcal{Q}_n$ with exchange coupling between nearest qubits (which have an edge between them). In addition, for having a switchable coupling an extra qubit between them is also needed [5]. The total Hamiltonian including both physical and ancillary qubits ($n2^{n-1}$) is given by:

$$\frac{H_{\mathcal{Q}_n}}{\hbar} = \frac{1}{2}\sum_{i=1}^{2^n} \omega_i \sigma_i^z + \frac{1}{2}\sum_{\langle i,j\rangle} \omega_{C_{ij}} \sigma_{C_{ij}}^z + \sum_{\langle i,j\rangle} g_i \sigma_i^x \sigma_{C_{ij}}^x + \sum_{\langle i,j\rangle} g_{ij} \sigma_i^x \sigma_j^x \tag{4}$$

We consider the dispersive regime in which the qubits are well detuned, i.e. $g_i \ll |\Delta_i| \quad \forall i$, with $\Delta_i = \omega_i - \omega_{C_{ij}} < 0$ the qubit-ancilla detuning. Therefore, we can use a perturbation theory in g_i/Δ_i. We use the Schrieffer-Wolff unitary transformation (SWT) $U_{SW} = e^{\eta}$. In our case,

$$U_{SW} = \exp\left(\sum_{\langle i,j\rangle}\left[\frac{g_i}{\Delta_i}\left(\sigma_i^+\sigma_{C_{ij}}^- - \sigma_i^-\sigma_{C_{ij}}^+\right) + \frac{g_i}{\Sigma_i}\left(\sigma_i^+\sigma_{C_{ij}}^+ - \sigma_i^-\sigma_{C_{ij}}^-\right)\right]\right) \tag{5}$$

After the SWT we end up with the effective qubit-qubit interaction Hamiltonian:

$$\frac{\tilde{V}}{\hbar} = \sum_{\langle i,j\rangle} \tilde{J}_{ij}\left(\sigma_i^+\sigma_j^- + \sigma_i^-\sigma_j^+\right) \tag{6}$$

where the effective *tunable* coupling between any two qubits is given by:

$$\tilde{J}_{ij} \approx \frac{g_i g_j}{2}\left(\frac{1}{\Delta_i} + \frac{1}{\Delta_j} - \frac{1}{\Sigma_i} - \frac{1}{\Sigma_j}\right) + g_{ij} \longrightarrow \frac{1}{2}\left[\frac{\omega^2}{\Delta\Sigma}\eta_{ij} + 1\right]\frac{C_{ij}}{\sqrt{C_i C_j}}\omega \text{ (for indentical qubits)} \tag{7}$$

The hopping term is tunable by setting the desired couplings and detunings. $\tilde{J}_{ij}$ can be altered negative when ancilla coupler frequency is reduced or changed to positive when this frequency is escalated. Therefore, we have some $\omega_{C_{ij}}^{\text{off}}$ such that $\tilde{J}_{ij}(\omega_{C_{ij}}^{\text{off}}) = 0$ within the bandwidth of each coupler. Thus, we obtain the switchable edges with $\omega_{C_{ij}}$ as the parameter.

Fidelity bound for imperfect implementation

The bound on the effective fidelity when the detuning parameters are not exact was analytically found as

$$\mathcal{F} \geq 1 - \sum_{m=1}^{\infty} \mathcal{O}(\|\mathcal{E}\|_F^m). \tag{8}$$

where $\mathcal{E}$ captures the effect of deviation from the ideal value. Using this bound, if maximum deviation in $\tilde{J}_{ij}$ for each edge is $\pm 0.5\%$, we have $\mathcal{F} > 97.43\%$ for the hypercube $\mathcal{Q}_4$.

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